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ADVANCED PARAMETRIC YACHT DESIGN

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Abstract. For high performance yacht design systematic investigations of the design space play an increasing role. Normally, numerical tank testing and race simulation programs are applied to find a set of near optimum designs which then are tested at model scale. Systematic shape variations, utilizing the gradient of the performance with respect to the selected design variables, can significantly accelerate the search for an optimum design, due to the reduction of designs to be evaluated. However, the gradual change of a hull shape is not a trivial task, especially if the design is governed by measurement rules. Performing the task of systematic shape variation with respect to a set of design variables manually is quite impossible. Taking into account the unique formulations of different measurement rules, no general tool can be applied. Instead a special design scenario has to be set up, where knowledge from designers, software engineers, sailors and the owner has to be merged to a single-objective formulation. This paper presents the course of an implementation, negotiations and the results of a parametric CAD-Program for a $30\,\mathrm{m}^2$ inshore yacht measurement rule (*German L-Boot*). Measurement restrictions, integrated knowledge from the designers experience and the definition of design variables are subject of an extensive discussion. The resulting modeller is fully based on parametric design principles and can be applied in an automated optimisation process without user interaction [1]. It is based on variational principles applying fairness measures, resulting in numerical battens. A tailor made set-up for the specific measurement rule and important parameters – identified in close co-operation with the designer – are made accessible and directly applicable for straightforward search of the optimum design.

NOMENCLATURE

$\vec{V}_{ij}$	Control point, i.e., vertex, of B-spline surface
B	Beam
D	Draft
E_n	Parametric fairness criteria for curve
$G(x)$	girth length
L	Length
$N_{ik}(t)$	B-spline blending functions
R_{wp}	Wave pattern resistance

1 INTRODUCTION

The current design practice of sailing yachts is a highly intuitive process and designers all over the world favor different approaches to make their specific design the most successful. The success of a yacht depends on the design of the hull, rig, sails and appendages, but also on the performance of the crew and, very importantly on the weather conditions during the actual period of the race. Specialised racing yachts are stuck to the single objective function of winning a particular race. Pleasure boats for racing and cruising purpose also have to match the owners complex requirements of personal desires and believes.

To generate a hull which incorporates all the requirements desired and restrictions mentioned above as a geometric model on the computer takes – depending on the complexity and the designers experience – between a couple of hours and some days. Once accomplished, scientific methods are available for design assessment with regard to resistance, seakeeping, rig forces, maneuverability, race simulation etc. and their results are reliable to a certain extend. The application of first principle methods in yacht design has so far been reserved to a limited number of researchers with lots of experience in their particular field of work.

Making changes of the design in order to improve its performance usually is a difficult task. It might take a couple of hours, and design constraints and the yachts performance have to be checked over again. Systematic shape variation are very time consuming, successful intuitive modifications gratify the quality of the designer involved in this task. This might also be a reason to classify a successful yacht-designer as an artist rather than a scientist.

If the design becomes a really complicated task, and the amount of money and prestige involved in a race campaign extends the usual, the believe into the designers intuition is just one part of the game. Therefore designers, meteorologists, naval architects, hydro- and aerodynamicists and other engineers are put together in teams for an optimum design procedure. Each of them adds his or her knowledge about one or several parts of the entire problem. The communication between the members of the design team represents the interfaces between the various programs each of them applies to perform a specific task. This communication, pre- and postprocessing, merging of results causes comparable slow compilation before a design can be assessed satisfactorily. Design variations can hardly be performed in a short time.

Recent publications by the authors [2, 3] have focused on the integration of parametric design of hulls and appendages and automated design assessment including formal optimisation on the basis of computational fluid dynamics and non-linear programming techniques. Design constraints have been treated as implicit constraints within the parametric design approach. However, the integration of measurement rules and owners requirements result in a

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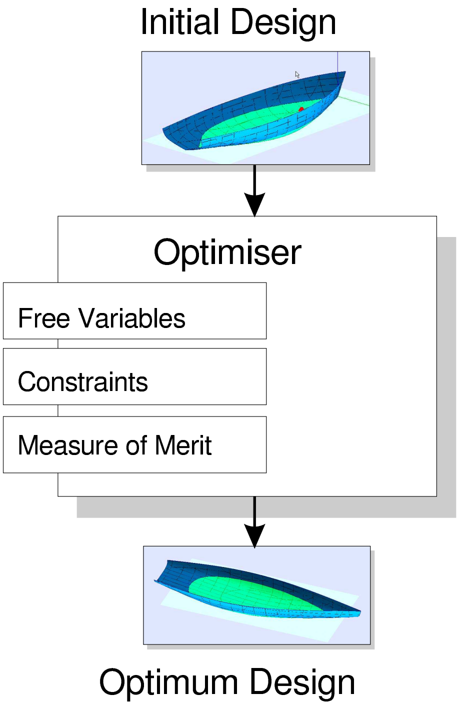


Figure 1. Components of an optimisation

rather complex set of constraints. While the formulation of the design problem is the difficult part, solving the mathematic can be done by common tools.

The applicability of formal hull shape optimisation has been proven and remarkable gains have been achieved. Fig. 1 shows the general scheme of an optimisation and its components. The effort put into the set-up of the formulation of the design problem as described above still produces costs one magnitude too high to be applied in a daily design routine. In order to develop a methodology to problem formulation in an acceptable period a joint project has been undertaken between the *Technical University Berlin*, *Carpe Diem Yacht Design* and *FRIENDSHIP-Systems*. As a case study, the problem formulation of a 30 m^2 inshore racing yacht has been selected including all relevant constraints. Subsequently an optimisation has been carried out and the feed-back from the designer and a potential customer was used for further refinement of the formulation. This paper describes the entire course of the project, its results and the lessons learned from it. It will demonstrate that custom made CAD programs are a kind of personalized spline for the designer who ordered it - he or she can concentrate on *what*, not on *how* in order to make better designs in a shorter time.

2 GATHERING INFORMATION

From the perspective of a software engineer – being used to turn information from articles, interviews or any other written or spoken word into an abstract set-up of objects, rules, relations and variables – gathering information is the most important part of the entire problem. Getting to know the problem is the key issue when setting up a model. In computer science, the model depicted in fig. 2 is well known, it points out the actions which have to be taken to produce a program representing existing or future processes and products.

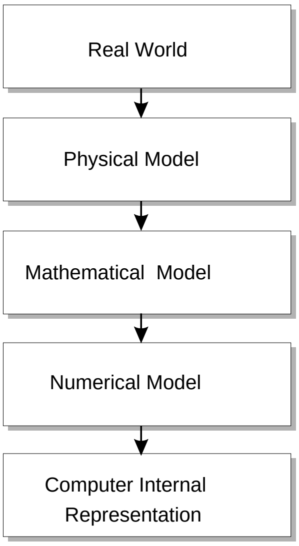


Figure 2. Steps from real world to CIR

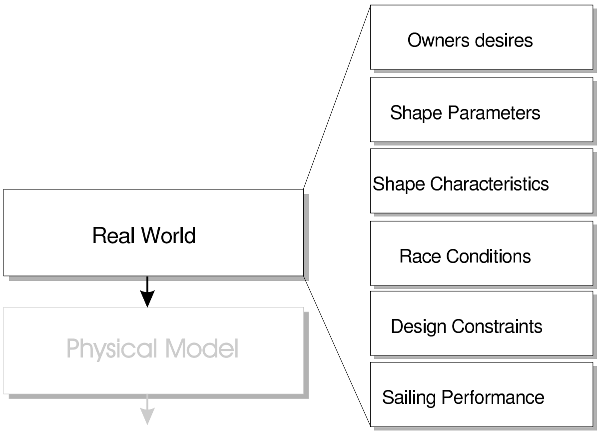


Figure 3. Sources for the real world important aspects

First of all one has to specify the real world, more precisely everything which seems to be important for the design and operation of the yacht, see fig. 3. The information collected for this task arises mainly from previous designs, measurement rules, reports and specifications from sailors and owners, and – most important – from the designers knowledge about typical design features.

The collection of data for the real world specification generates some paperwork, which is similar to the conventional design work, although the collector of data is not the designer. First of all the measurement rules are to be examined very thoroughly by the software engineer. Subsequently the co-operating designer has to put in all known aspects of the design. For most measurement re-

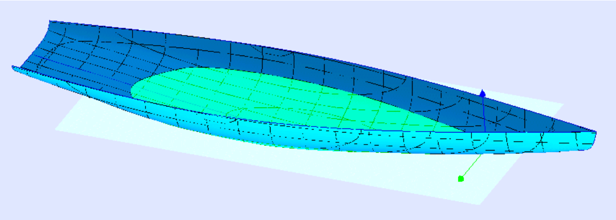


Figure 4. Perspective view of a 30 m^2 yacht hull

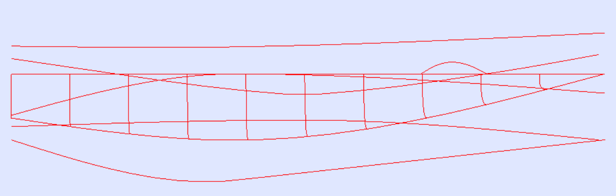


Figure 5. Basic curves of a 30 m^2 yacht

stricted boat classes typical characteristics have developed during the past: e.g. successful racing yachts have always been built as beamy as allowed, possibly as light or as heavy as limited by the rule; steps or knuckles right behind or ahead of measurement marks to reduce freeboard and weight, discontinuities for better rating values – these information have to be supplied by experienced designers. This evolutionary optimisation which has already taken place before the current challenge, might reduce the number of unknowns and supplies reasonable starting points for further design work.

For the project presented here, the initial data was made available from

1. the original measurement formula including later modifications,
2. one very early linesplan,
3. two different 3D CAD-models of already built hulls,
4. the designer of the two previous designs and
5. a potential producer of a small series of this type of yacht.

From the data collected, a first parametric model was established and implemented similarily to what was demonstrated by the authors in earlier publications, see [2, 3]. The parametric model was presented to the designer and the model was refined until a satisfying geometry had been achieved.

3 PARAMETRIC MODEL

The parametric modelling approach developed by the authors consists of multiple nested optimisations of uniform B-spline curves. The intuitive properties of B-spline curves makes them the favorable type of representation for most modelling tasks in naval architecture. For B-spline curves, the definition is given by:

$$\vec{Q}(t) = \sum_{i=0}^{m-1} \vec{V}_i N_{ik}(t) \tag{1}$$

where m is the number of vertices, k the order of the curve, $\vec{V}_i$ are the vertices of the defining polygon and N_{ik} are the base polynomials each representing the influence of a vertex within a certain range of the curve. A detailed description can be found in [4]. A set of longitudinal B-spline curves represent the properties of the hull shape, see fig. 5. For instance the plan view of the deck can be specified by its forward end at the bow, its coordinates of the maximum beam and at the transom. Within the *FRIENDSHIP-Modeler* a B-spline curve is then generated from that given information which optimises the shape of the curve in respect to optimum fairness, i.e., a numerical algorithm arranges the vertices in such a way that the specified properties are met while the curve is kept as fair as possible. As a measure of fairness, the integral of the second derivatives of the B-spline curve can be easily applied:

$$E_2 = \int_0^1 (\frac{dx^2}{d^2t} + \frac{dy^2}{d^2t} + \frac{dz^2}{d^2t})dt \tag{2}$$

The implementation of the program allows additional for properties which can be user specified within the parametric definition. The parameter file – a human readable ASCII file format – can be supplemented by numerous optional parameters to further refine the shape. Since the parameter file is runtime interpreted, the modified optimisation problem is set up anew and the shape will consider possible new constraints. The methodology has been described by of the authors in depth, see for example [5].

The longitudinal descriptor for the shape of the deck has been selected as one example. Other sectional properties are mandatory for the hull definition, some are optional. For conventional round bilge hull shapes it should be sufficient to describe the plan view of the deck, the sheer-line, flare at deck and deadrise along the centerplane curve, which is also required. From this set of curves, see fig. 5, sections can be created at any longitude. From a set of sections, subsequently a hull surface can be generated by interpolation the sections. The uppermost curve depicted in fig. 5 shows the sheer line, also the deck plan view and the center plane curve can be identified.

The parametric description allows to fulfil a number of design constraints which shall be called implicit constraints. The modelling approach allows direct access to form parameters like maximum beam, freeboard at different positions, draft and convexity of selected plane sections. Also differential and integral parameters can be implemented as implicit constraints, e.g. curvature, lateral area and volumetric properties. Naturally, not all constraints and measurement formulations can be realised implicitly within the geometric description. However, the

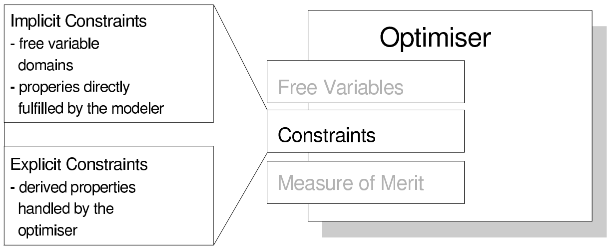


Figure 6. Classification of constraints

more constraints can be fulfilled directly, the easier the entire problem formulation can be applied in an automated optimisation. The handling of explicit constraints is described in the following section.

4 CONSTRAINTS

Several aspects have to be considered when implementing constraints in an automated design procedure. A classification of constraints is depicted in fig. 6. A distinction has to be made between implicit and explicit constraints. Implicit constraints can be set within the geometric generation process and are not violated for any design alternative.

Examples for constraints which can be formulated implicitly:

1. the feasible domain of design parameters of a parametric description, e.g. draft or beam of the hull, sail area, luff length, displacement etc.
2. parameter ratios can be set up to fulfil single or sets of constraints relative to each other, e.g. beam vs. length ratio,
3. constant parameter values – the knowledge gathered from previous designs result in parameters which are kept constant, e.g. the freeboard at a specific location is always as low as allowed by the rule, even though it would be within the rule to range from one value to another.

Explicit constraints mainly consist of derived properties. The basic difference between implicit and explicit constraints in the presented approach can be read as follows: implicit constraints are fulfilled directly at the stage of producing the geometry, explicit constraints have to be investigated afterwards. E.g. the girth length at a specific section must not be greater than a certain value. Since the girth length in the presented approach results from sectional properties like beam at deck, tangents, draft and curvature constraints, the function of the girth length is a function of several design parameters.

$$G(x=M_s)=\int_0^1(\vec{Q}(u,v)\cap M_s)ds \tag{3}$$

Table 1. Measurement constraints of the L-Boot

lower bound	parameter	upper bound	type
1.4	$\frac{L_{max}}{L_{WL}}$	1.6	explicit
$1.0\,t$	displacement		explicit
$1.75\,m$	B_{max}		implicit
$0.36\,m$	$freeboard$	$0.4\,m$	implicit
	$\frac{L_{max}}{B_{max}}$	4.5	implicit
	$\frac{overhang_{Bow}}{overhang_{Total}}$	0.6	explicit
	$\frac{overhang_{Stern}}{overhang_{Total}}$	0.6	explicit
0.9	$\frac{B_{WL}}{B_{max}}$		explicit

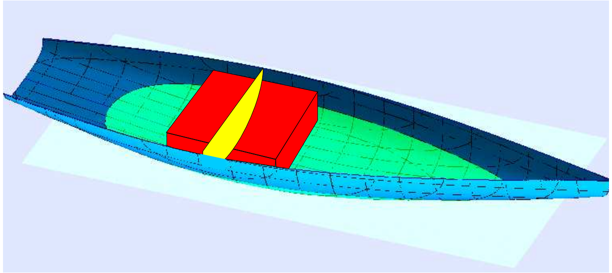


Figure 7. Dominating constraints for the principal dimensions

M_s denoting the measurement section, and $\vec{Q}(u,v)$ the $3D$ representation of the hull surface. The intersection between the surface $\vec{Q}(u,v)$ and the section M_s with $x = const.$ must be determined and the arc length of the intersecting curve calculated.

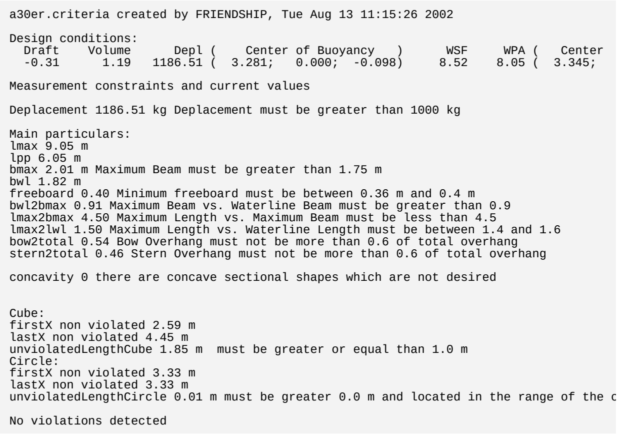
As mentioned above, the question if a constraint is implicit or explicit does not have a unique answer, it is a matter of the formulation of the design description.

The measurement rule of the $30\,\mathrm{m}^2$ L-Boot contains a number of design constraints which are listed in tab. 1.

Some of the explicit constraints listed in the table can be converted into implicit constraints, which has also been done in the course of the implementation. Controlling the ratio of waterline length vs. total length can be done directly, if the waterline length and the ratio itself are turned into free variables with a limited feasible domain for the latter one.

However, additional constraints which dominate the principal dimensions of the hull have not been mentioned yet:

A box, measured $1000\,\mathrm{mm} \times 1000\,\mathrm{mm} \times 550\,\mathrm{mm}$ $(L \times B \times D)$ located with its upper side at minimum freeboard elevation must fit inside the hull. In addition, a half circle lying in sectional plane, sized 700 mm in radius with



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Design conditions:
  Draft    Volume      Depl (    Center of Buoyancy   )      WSF     WPA (   Center
  -0.31      1.19   1186.51 (  3.281;   0.000;  -0.098)     8.52    8.05 (  3.345;

Measurement constraints and current values

Deplacement 1186.51 kg Deplacement must be greater than 1000 kg

Main particulars:
lmax 9.05 m
lpp 6.05 m
bmax 2.01 m Maximum Beam must be greater than 1.75 m
bwl 1.82 m
freeboard 0.40 Minimum freeboard must be between 0.36 m and 0.4 m
bwl2bmax 0.91 Maximum Beam vs. Waterline Beam must be greater than 0.9
lmax2bmax 4.50 Maximum Length vs. Maximum Beam must be less than 4.5
lmax2lwl 1.50 Maximum Length vs. Waterline Length must be between 1.4 and 1.6
bow2total 0.54 Bow Overhang must not be more than 0.6 of total overhang
stern2total 0.46 Stern Overhang must not be more than 0.6 of total overhang

concavity 0 there are concave sectional shapes which are not desired


Cube:
firstX non violated 2.59 m
lastX non violated 4.45 m
unviolatedLengthCube 1.85 m  must be greater or equal than 1.0 m
Circle:
firstX non violated 3.33 m
lastX non violated 3.33 m
unviolatedLengthCircle 0.01 m must be greater 0.0 m and located in the range of the c

No violations detected
```

Figure 8. Constraints and principal dimensions listed in criteria file

its center at the same elevation as the minimum freeboard must also lie inside the hull. The longitudinal position of the circle must be located within the range of the box, as depicted in fig. 7.

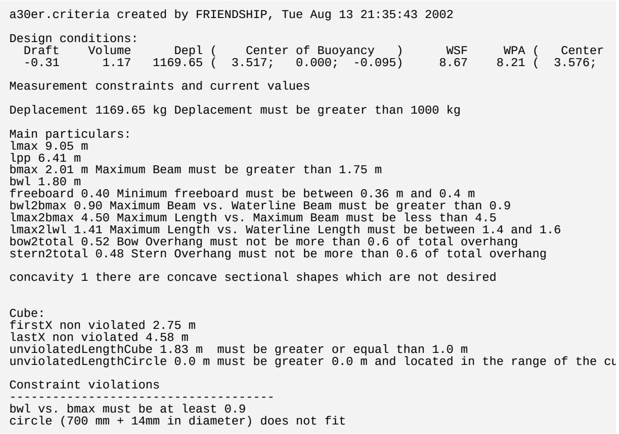
From the previous designs it was rather obvious that the inner box constraint would not be critical, but the circle was the one which tended to be violated.

The handling of explicit constraints is a matter of the optimisation algorithm, or alternatively of a pre-processor checking each design generated by the set of design variables for validity. Within the approach selected by the authors, the handling of explicit inequality constraints is part of the optimisation algorithm. For each design a corresponding file is generated which contains relevant data for design assessment. Beside the values needed for constraint control, hydrostatic data is provided and design violations are listed in a separate paragraph which makes it also a useful tool for manual design work.

To demonstrate the simplicity and obvious advantage of the automated constraint control, two parameters have been changed slightly to induce constraint violations. Fig. 9 shows the resulting criteria file displaying the violations at its bottom.

5 INITIAL DESIGN

For the initial shape one of the available previous designs – the better rated one – has been chosen and modelled parametrically. Having the most probable racing conditions in mind, slight modifications have been introduced to reduce the wetted surface area as the low wind condition dominates the races. Since the constraints were easy to control, it was no problem to find a hull, which fulfilled the constraints and featured about 10 % less wetted surface, a big advantage for the specified objectives. Having an extremely short design waterline, the wave resistance was expected to be relatively high. From earlier optimisations it was known that by hull variations on the basis of the parametric design methodology, it was rather easy to reduce



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Design conditions:
  Draft    Volume      Depl (    Center of Buoyancy   )      WSF     WPA (   Center
  -0.31      1.17   1169.65 (  3.517;   0.000;  -0.095)     8.67    8.21 (  3.576;

Measurement constraints and current values

Deplacement 1169.65 kg Deplacement must be greater than 1000 kg

Main particulars:
lmax 9.05 m
lpp 6.41 m
bmax 2.01 m Maximum Beam must be greater than 1.75 m
bwl 1.80 m
freeboard 0.40 Minimum freeboard must be between 0.36 m and 0.4 m
bwl2bmax 0.90 Maximum Beam vs. Waterline Beam must be greater than 0.9
lmax2bmax 4.50 Maximum Length vs. Maximum Beam must be less than 4.5
lmax2lwl 1.41 Maximum Length vs. Waterline Length must be between 1.4 and 1.6
bow2total 0.52 Bow Overhang must not be more than 0.6 of total overhang
stern2total 0.48 Stern Overhang must not be more than 0.6 of total overhang

concavity 1 there are concave sectional shapes which are not desired


Cube:
firstX non violated 2.75 m
lastX non violated 4.58 m
unviolatedLengthCube 1.83 m  must be greater or equal than 1.0 m
unviolatedLengthCircle 0.0 m must be greater 0.0 m and located in the range of the cu

Constraint violations
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bwl vs. bmax must be at least 0.9
circle (700 mm + 14mm in diameter) does not fit
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Figure 9. Violated constraints in criteria file

wave resistance due to displacement redistribution by the magnitude of 10 % to 20 %. It was also felt that it would be favorable to keep the displacement as low as possible to give good speed at light wind speeds and downwind.

First CFD-calculations gave a good impressions of the magnitude and relation of wave and frictional resistance, which again confirmed the selected approach:

- implement design characteristics for the expected racing conditions
- in this case minimize wetted surface
- optimise for total resistance for the upper range of speed
- select free variables which enable the design to redistribute displacement, (see next section).

6 DESIGN VARIABLES

The experience from previous hydrodynamic designs based on non-linear potential flow assessment indicated the use of certain design variables. In addition, the design presented here did not only have to be fast, but also had to fulfil some aesthetic aspects, which were important for the sales manager of the yard, due to the typical characteristics of the L-Boot. The first optimisations carried out without taking those characteristics into account, turned the bow of the yacht into a modern type of hull with much reduced deadrise in the forward part.

However, a challenge for the set up of the design formulation and the selection of design variables was related to the dominating explicit constraints – i.e. the half circle – and the obvious tendency of the hull to get faster with reduced displacement. The requirement of the waterline to maximum beam ratio, see tab. 1 also called for a non-standard parameter, which was applied in the optimisation. Two additional parameters where employed to match the requirements and enable optimum shapes for the desired objective function. One parameter controls the initial speed of the vertical surface parameter, which corresponds to the curvature of the hull starting at deck level pointing downwards. Even for very shallow section shapes one can produce shapes with a beamy waterline. The second parameter controls the sectional area which also supports the hulls desire for reduced displacement. The parametric control of the sectional area curve facilitates a longitudinal shift in displacement – a powerful dial with respect to wave resistance. Due to the large overhangs, typical of this type of yacht, the sectional area control was not restricted to the still water submerged part, but was applied for the full length of the hull.

Several optimisation runs have been performed with varying sets of free variables and their range. The change of design variables was not done due to missing effects on the performance, but due to changes in the shape characteristic, which the designer judged not acceptable by a potential owner of the yacht. Beside the parameters above, several very effective design parameter shall be mentioned here. The position of the maximum draft and beam, the transom beam, deadrise in the bow region and the radius of the centerplane curve at its maximum draft position, the latter having pronounced effect on the total displacement. Each run was performed with about 10 free variables.

7 OPTIMISATION

Upright total resistance was selected as the objective function applied for the hull optimisation. The light wind race conditions gave an estimate of five to ten degrees and at maximum 20 degree of heel. The design assessment was performed applying *SHIPFLOW*, the well known potential non-linear free surface CFD code, see [6, 7]. The hull was allowed to trim and heave freely, which seemed to be very important because of the immense bow and stern overhangs. Earlier research has proven the reliability of the relative results, thoroughly discussed in [8].

The extended set of explicit constraints lead to the choice of the tangent search method, a classic search algorithm being able to deal with inequality constraints, see [9]. Based on a modified Hooke-Jeeves algorithm, a search direction is obtained from a local deterministic search. Constraint violations are recognized and the search direction is corrected in order to stay within the feasible domain. The algorithm searches along the tangent of the violated constraint, which reduces the number of function evaluations significantly, compared to a simple deterministic search where constraint violations simply result in invalid designs or penalties.

Fig. 12 shows the lines plan of the base hull for the initial optimization run and fig. 13 shows an intermediate result after 99 variations. While wavemaking resistance according to *SHIPFLOW* and an approximation for the frictional resistance for the base configuration summed up to 216.5 N for a speed of 3 m/s the optimization yielded an improvement of 10 % (194.9 N) after generating 99 hulls of which 65 fulfilled the measurement constraints and were evaluated by *SHIPFLOW*. Altogether the optimization for this simple configuration took about 12 hours on a contemporary desktop computer.

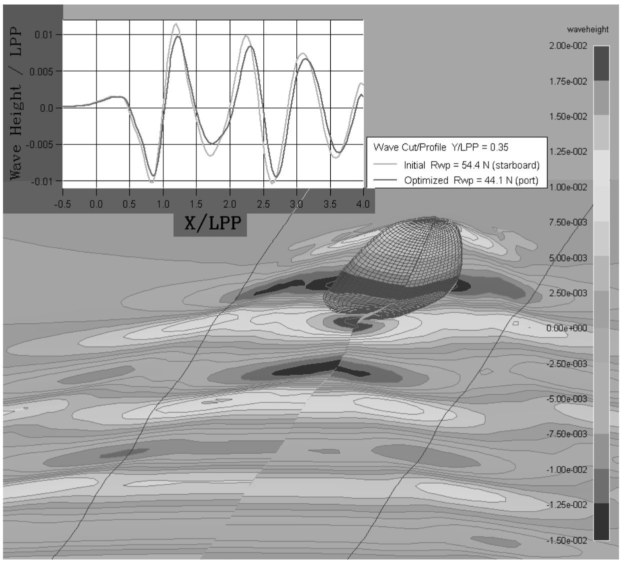

Figure 10. Wave pattern comparison of intital (starboard) und optimised (port) hull, (R_{wp} denotes the wave resistance obtained from wave pattern analysis.)

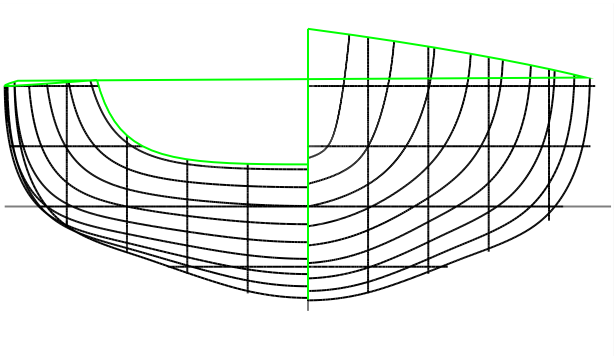

Figure 11. Concave sections in the midship section range

The calculation of wavemaking resistance by pressure integration – as done by *SHIPFLOW* – is known to have a strong dependency on panelization. Therefor a wave pattern analysis by [10] was undertaken. It confirmed the gains computed from pressure integration.

Fig. 10 shows a comparison of the inital and the optimized hull wave patterns.

After the automated optimisation the parameter file was sent to the designer who did some more modifcations especially above the waterline to improve the aesthetic acceptance by the customer. Finally, the influence of the manual modifications to the performance was checked numerically and discussed with the designer.

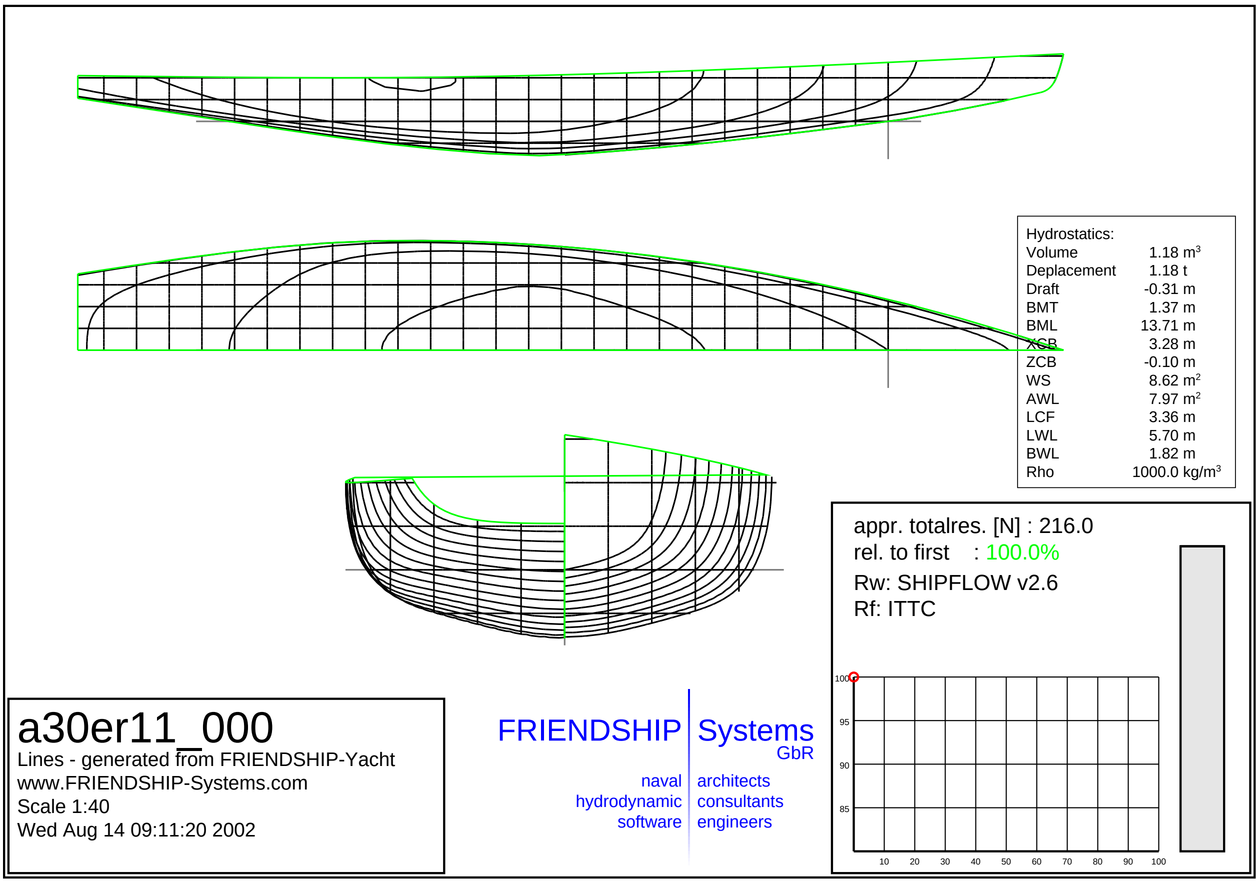

Figure 12. Lines of base hull

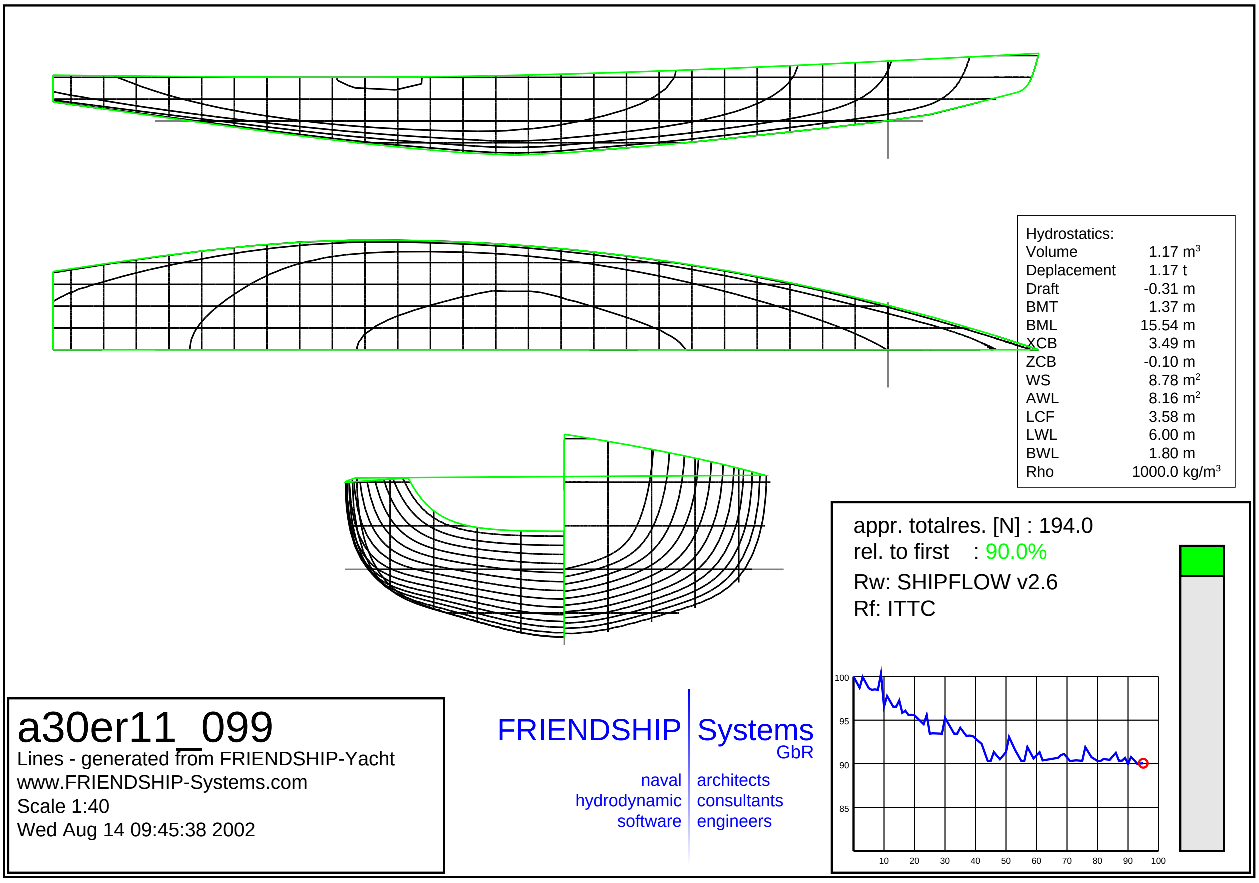

Figure 13. Lines of optimized hull and optimization history

8 ITERATION

Presenting the best results of the optimisation, the hull showed better performance over the entire range of interest than the previous designs. The potential owner, however, was still not satisfied with the shape from an aesthetic point of view. The resulting shape encloses the circular arc at just one single position. In addition the displacement is reduced by the local reduction of the sectional area in that region. The displacement reduction generates sections that are concave in the region around the maximum draft of the canoe body, see fig. 11. Negotiating that such a hull can not be sold easily an additional constraint was introduced. From a mathematical point of view the concave sectional shapes can be detected calculating the curvature of the frames. The convexity constraint is quite well known from the IACC measurement rule and has been applied successfully in the subsequent optimisation, even though just as a sectional criterion.

During the course of setting up the problem formulation, reviewing results and more or less desirable shapes with the designer and owner, a few new features were added to the model. As a result of the iterative implementation a tailor-made modeler has been developed up, which definitely fulfils all formal constraints of the measurement formula, but not necessarily the desires of a sales manager or a potential owner. As mentioned earlier in the paper, the implemented modeler does not replace any specialist, but provides a personalized modeling tool, in this case with emphasis on speed and aesthetics, still leaving options for pushing the speed.

9 CONCLUSION

The set-up of complex design problems fr yacht optimisation has been shown to be successful. It requires special knowledge on design parameters from a mathematical point of view.

The set-up of the design formulation took about two man months, which is far too expensive for a private, standard sailing yacht design. If this methodology shall be applicable for daily design work, either draw backs in the detail have to be acceptable, the number of iterations have to be limited or the procedure to formulate the design problem have to be standardised, which took most of the time for this optimisation work. The communication between all partners involved has to be performed on the basis of standard procedures which have to be further developed. For high performance yacht design, e.g. for prestigious races, the set-up of the problem formulation can be realised in a couple of person months.

ACKNOWLEDGEMENT

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