Constraint Management for Marine Design Applications

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Abstract

The design of marine systems is often dominated by a considerable number of constraints which are related to the many competing aspects pertinent to the product's life cycle. The work presented here originates from automated optimization in which constraints are handled in a formal and unified manner. An analogous approach is proposed for systematic design space exploration on the basis of parametric design principles. This allows comprehensive investigations in which constraint monitoring and assessment bring about valuable insight. The constraint management is illustrated on the basis of a representative hull design project.

Keywords

CAD; Constraint; Geometric modeling; Hull design; Optimization; Ship design.

Introduction

Designing an object of high complexity implies that quite a few constraints need to be satisfied while several opposing objectives ought to be maximized (or minimized). This makes design a creative and interesting undertaking but also a challenge if many constraints are imposed. Both constraints and objectives usually are of multi-disciplinary character and may stem from hydrodynamics, structures, production, operation, economics etc.

Design constraints are typically controlled by the person being responsible for a certain task. Many constraints result from previous or subsequent tasks carried out by different team members or departments. Feed-back is given only if one or more constraints are violated. The cure to constraint violation is manifold: Ideally, the constraint in question can be slightly relaxed. More typically, however, a compromise needs to be negotiated which means that a loss of performance has to be accepted or a nested change of many design aspects has to be brought about. In order to make a rational and well-founded decision comprehensive knowledge about the design space, the objectives and the constraints as well as their dependencies on the design variables is mandatory. Questions that are usually asked are:

- In view of the many constraints, are there any feasible designs at all?
- The existence of feasible designs presumed, how close are favorable designs to one or several constraints?
- Which constraints are dominant and which might have no tangible influence (and can thus be discarded)?
- Which free variables are important and which are insignificant with respect to both the objectives and the constraints?

If those questions can be answered the number of iterations caused by constraint violations will decrease while the quality of the design will increase.

This paper therefore attempts to address important issues of constraint management. Firstly, a theoretical view on the ideas and concepts is presented so as to introduce instruments of constraint assessment. Secondly, a practical application is given to illustrate the feasibility and the merits of the approach. The example considered was realized within *FRIENDSHIP-Systems*' optimization framework applying the fully parametric CAD tool *FRIENDSHIP-Modeler*.

Constraint classification

Formal optimization

In an optimization a good (or the best) solution is determined from a set of alternative and feasible solutions. The best solutions – i.e., the Pareto optima – are selected in terms of the free variables on the basis of problem-oriented criteria. The standard format of mathematical programming is as follows, see Birk and Harries (2003):

Free variables: They are the independent decision variables which uniquely describe the problem $\vec{x}^T = (x_1\ x_2\ \dots\ x_i\ \dots\ x_n)$.

Objectives: They are the criteria by which a solution is assessed. A criterion is a function of the free variables $F(\vec{x})$.

Constraints: They reduce the possible combinations of the free variables to the set of feasible combinations. Several types are distinguished:

- Bounds $x_{i\,min} \leq x_i \leq x_{i\,max}$ for $i = 1, \ldots, n$.
- Equality constraints $h_j(\vec{x}) = 0$ for $j = 1, \ldots, m$.
- Inequality constraints $g_k(\vec{x}) \leq 0$ for $k = 1, \ldots, p$.

This formal treatment of free variables, objectives and constraints has the key advantage that similar strategies can be utilized for many optimization problems independent of their nature. Consequently, it is advantageous to employ this unified view also for constraint management (even though no automated optimization might actually be intended).

Constraints in geometric modeling

There are many different types of design constraints which relate to all possible aspects of a product. Many constraints are of geometric nature or can be ascribed to geometric properties. The authors' background being hydrodynamic design of ship hull forms, representative constraints shall be discussed in the context of geometric modeling.

Three types of geometric constraints can be distinguished:

- Differential,
- Positional,
- Integral.

Typical positional constraints are maximum dimensions (e.g. breadth must be no greater than ...), collision control (e.g. the hull must remain outside a virtual volume to guarantee minimum propeller clearance) and hard points (e.g. the engine foundation or a container must obviously not penetrate the hull and its inner structure). An example for collision control is shown in Fig. 1 while a hard point constraint can be seen in Fig. 2. A detailed example may be found for instance in Hochkirch et al. (2002).

Examples for integral constraints are volume (e.g. the displacement volume is between ..., the tank volume equals to ...) and centroids (e.g. the longitudinal center of buoyancy shall shift no more than ...) which constitute direct geometric constraints. Indirect integral constraints often originate from hydrostatic considerations (e.g. minimum metacentric height as determined via the geometry of the design waterline).

Differential constraints are fairness (e.g. no hollowness must be present in a certain hull region) and producibility (e.g. plates should be developable to a given extend) etc. As opposed to positional constraints which usually are straight forward to evaluate, differential constraints call for more computational effort and, possibly, even require some additional simulations.

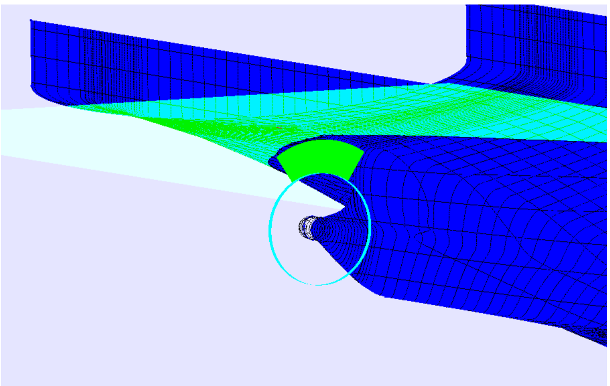

Figure 1: Feasible hull shape with regard to propeller clearance

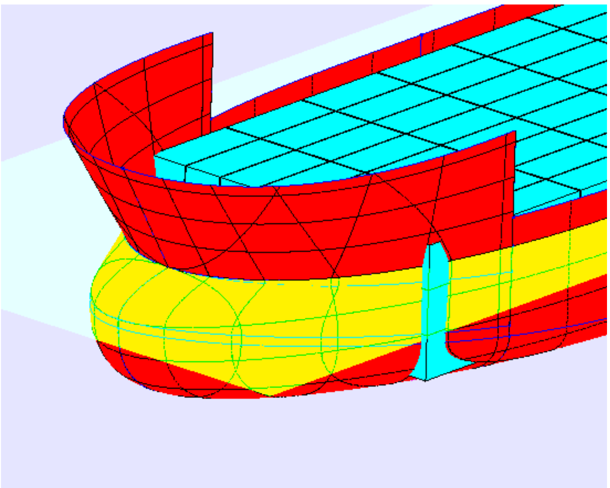

Figure 2: Infeasible hull shape due to violation of container hard points

As becomes evident from the examples above many constraints are of inequality type and can generally be written in the form

$$C_k(\vec{x}) \leq C_{k_{Max}} + \tau_k \quad (1)$$

or

$$C_k(\vec{x}) \geq C_{k_{Min}} - \tau_k \quad (2)$$

and, hence, converted into the standard format

$$g_k(\vec{x}) := C_k(\vec{x}) - C_{k_{Max}} \leq \tau_k \quad (3)$$

and

$$g_k(\vec{x}) := C_{k_{Min}} - C_k(\vec{x}) \leq \tau_k \quad , \quad (4)$$

respectively. Here τ_k is a user-specified tolerance. It equals 0 under strict circumstances but may assume a small positive value if a constraint violation could be acceptable to investigate the region just outside the feasible domain.

In geometric modeling equality constraints are of less importance since they often serve to reduce the number of free variables, see for instance Abt et al. (2001) for more details. Therefore, priority is given here to the management of inequality constraints.

Constraint management

Constraint management comprises

- handling (set up),
- analysis,
- monitoring,
- assessment.

The handling and analysis of constraints is realized inside the Computer Aided Design tool or outside – the latter for instance if an elaborate simulation needs to be performed such as a Finite Element Method analysis (FEM) or a Computational Fluid Dynamics run (CFD). In order to monitor and assess a set of constraints it is necessary to decide on a set of free variables $\vec{x}$ which might eventually influence the design – similar to what is done in an automated optimization. A parametric approach to geometric modeling should be followed so as to reduce the design's complexity on the basis of a problem-dependent high-level definition, see Birk and Harries (2003).

Monitoring and assessment

Trying to circumvent a potential bias, a (quasi-)random sequence of variants should be generated which can be achieved by applying a Sobol algorithm, see Press et al. (1988). Fig. 3 illustrates the distribution of a Sobol sequence with 5000 variants. The actual design problem comprised 14 free variables. A plot in $\mathbb{R}^{14}$ being impossible, projections onto the $\tilde{x}_1$-$\tilde{x}_2$-plane and the $\tilde{x}_1$-$\tilde{x}_3$-plane have been selected, $\tilde{x}_1$, $\tilde{x}_2$ and $\tilde{x}_3$ being normalized free variables

$$\tilde{x}_i := \frac{x_i - x_{min}}{x_{max} - x_{min}} \quad . \tag{5}$$

Any other combination of free variables would result in similar figures.

The Sobol sequence constitutes a design of experiments (DoE) and brings about the statistical basis for monitoring and assessment. (It is also regularly performed early in an automated optimization for exploration purposes, see Abt et al. (2003). The key difference is that computational intensive objectives need not be computed at this stage.)

A first assessment of the complexity of the design domain is to compute the ratio between the number of feasible designs N_f and the overall number of designs N_t:

$$d := \frac{N_f}{N_t} \quad . \tag{6}$$

An unconstrained problem is then characterized by $d = 1$ while a fully active problem shows $d = 0$. A weakly constrained problem may be found if $0.9 < d < 1.0$ while

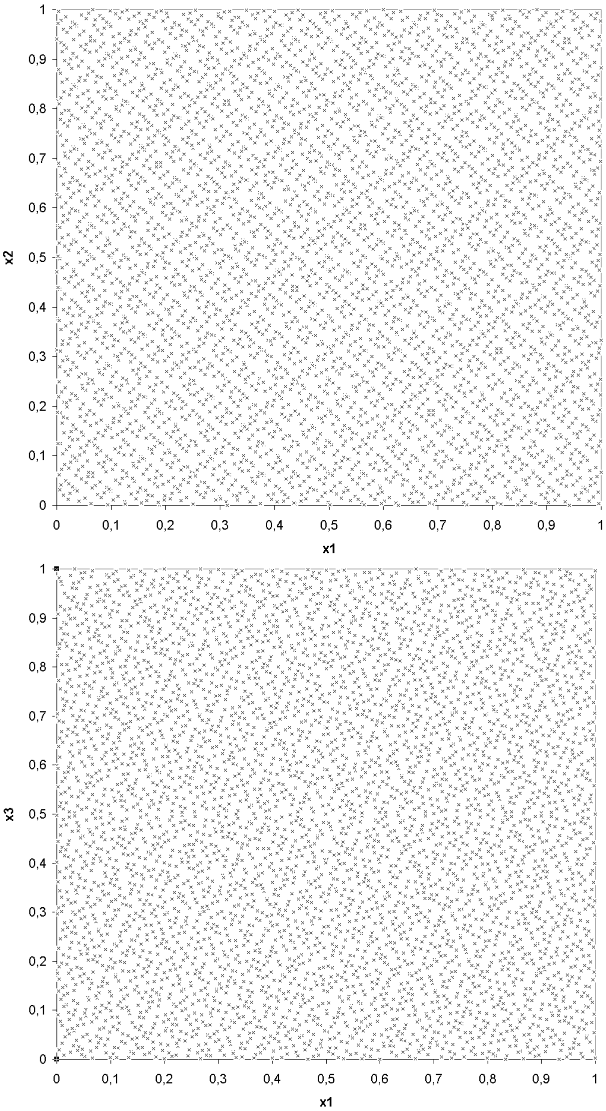

Figure 3: Sobol sequence

a constraint dominated problem is given by $0 < d < 0.1$. (This is a subjective appreciation which may shift according to the problem field.)

If the domain index is rather small – i.e., just a few designs are valid – it is useful to contemplate the relaxation of one or several inequality constraints. The domain index d varies with the limiting values $C_{k_{Min}}$ and $C_{k_{Max}}$ of all constraints, see Fig. 6 and further discussion below. The variation of d with respect to changes in the specified limiting values therefore provides information on which constraints are beneficial to relax so as to gain an increased number of feasible designs. Since minor changes are more likely to be acceptable high partial derivatives close to the original limits are advantageous because even a small constraint relaxation will then already help.

When assessing specific constraints it is rather straight forward to distinguish active ($g_k(\vec{x}) \geq 0$) and inactive ($g_k(\vec{x}) < 0$) constraints. However, it is less obvious (and much more interesting) to judge the extend by which a constraint is violated or how much leeway there still is in the design. A utilization index shall therefore be proposed

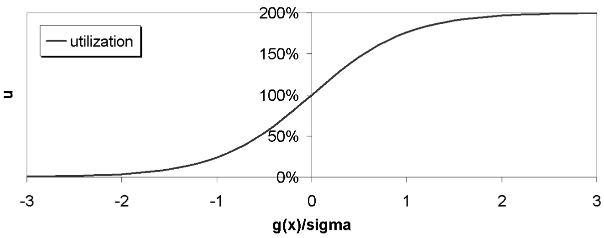

Figure 4: Constraint utilization index

for quantification:

$$u_k := \tanh\left(\frac{g_k(\vec{x})}{\sigma_k}\right) + 1 \quad . \tag{7}$$

The utilization index u_k is a function of the normalized k-th constraint $\frac{g_k(\vec{x})}{\sigma_k}$ where σ_k denotes the root of the variance of the frequency distribution

$$f_k = f_k\left(\frac{g_k(\vec{x})}{\sigma_k}\right) \tag{8}$$

as depicted in Fig. 5.

Fig. 4 shows the utilization function $u_k\left(\frac{g_k(\vec{x})}{\sigma_k}\right)$. Important properties are that it is strictly monotonic and that $u_k(0) = 100\%$ for $\frac{g_k(\vec{x})}{\sigma_k} = 0$ (i.e., where a design lies on the inequality constraint itself). Moreover, $u_k(-\infty) = 0\%$ for designs which lie far away from the constraint while $u_k(\infty) = 200\%$ for designs which heavily violate the constraint. Finally, the slope of u equals 1 at $\frac{g_k(\vec{x})}{\sigma_k} = 0$ which yields an almost linear dependency in the vicinity of the constraint just becoming active.

Fig. 5 shows the frequency distribution f_k of a Sobol sequence with respect to the k-th constraint g_k. σ_k is used to normalize all terms so as to standardize the frequency distribution and to achieve independence of the actual constraint values. μ_k denotes the mean.

A frequency distribution as depicted in Fig. 5 suggests that the constraint g_k is non-critical for the majority of variants. (Of course, it needs to be kept in mind that an optimization performed at a later point in time could still lead into a region in which g_k becomes active.) Each variant can now be assessed in terms of its utilization index. In addition, the utilization index at the mean $u_k\left(\frac{\mu_k}{\sigma_k}\right)$ and at other prominent values such as $u_k\left(\frac{\mu_k}{\sigma_k} \pm 1\right)$ give an idea of the constraint's severity. (The utilization index might also be serviceable to get an idea about a designs robustness. If two designs perform equally well, the one with smaller utilization index may be the better.)

Illustrating example

The constraint management as theoretically discussed above shall now be illustrated on the basis of a practical example taken from a contemporary ferry design project. A total of 13 inequality constraints were considered. The

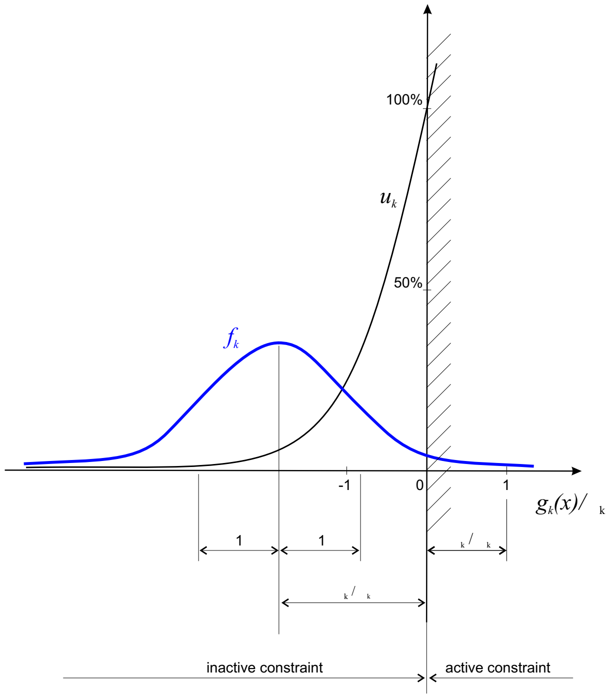

Figure 5: Frequency distribution and utilization index

constraints were of positional and integral type. Differential constraints on fairness were accommodated implicitly by means of an inner geometric optimization as realized with the *FRIENDSHIP-Modeler*, see Harries (1998).

The first set of inequality constraints was so confining that just a few feasible designs were present in a Sobol investigation of 5000 variants, the domain index as introduced above being $d = 2\%$. Those initial candidates, however, were considered unfavorable with respect to hydrodynamic performance (which in the end had to be optimized). For instance, it turned out that GM_T was much higher than desired. An increase in the number of feasible designs could only be achieved by relaxing some constraints.

In order to identify constraint limits which are beneficial to modify (if permissible) the domain index d is studied. Fig. 6 shows d for a variation of two selected constraints – one on the ferry's draft and another on its freeboard. For the actual values of $C_{draft_{Max}}$ and $C_{freeboard_{Max}}$ about 50% of the designs were feasible. Increasing $C_{freeboard_{Max}}$ features a strong dependency of d which means that a small relaxation of the limiting value already yields a substantial rise in the number of feasible hull forms. Meanwhile, slight changes in $C_{draft_{Max}}$ do not bring about any tangible freedom, the slope being almost horizontal. Consequently, it appears to be more profitable to consider the relaxation of the freeboard constraint if need be.

In addition, it can be observed in Fig. 6 that both curves assume horizontal branches on either side. When tightening the limiting values the number of feasible designs naturally drops to zero. When relaxing a constraint, however, one or several other constraints become active at some point.

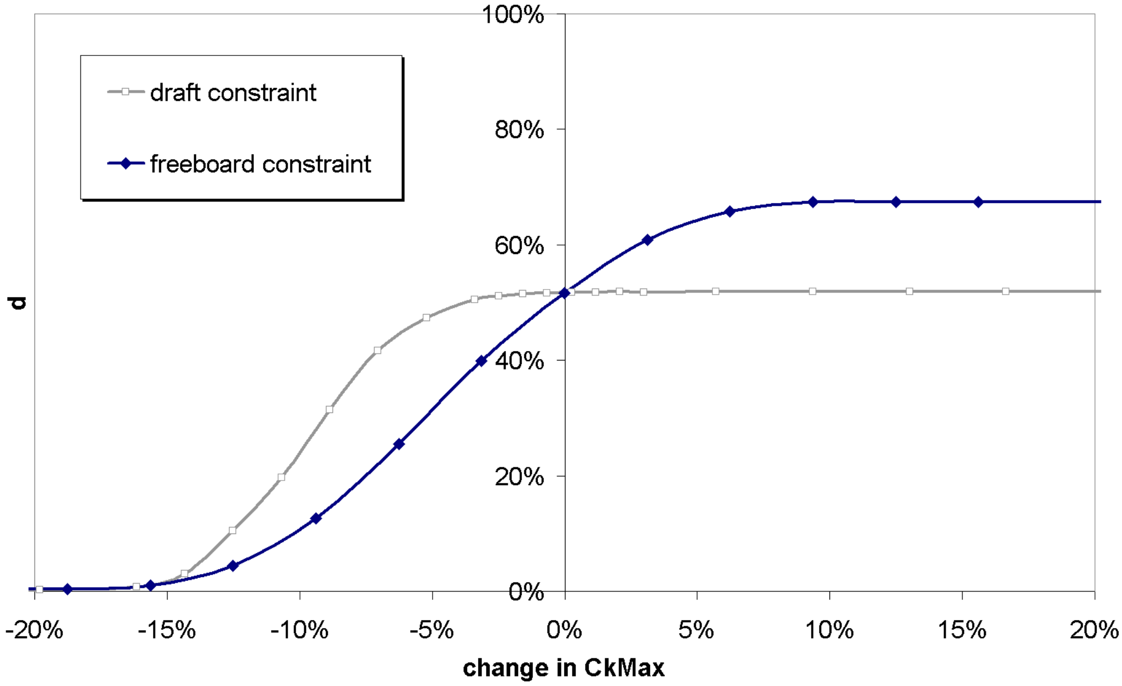

Figure 6: Domain index for two constraints

A further assessment of the two example constraints on draft and freeboard is undertaken on the basis of their frequency distributions, see Fig. 7 and Fig. 8. It can be seen that the bulk of the designs is rather close to the freeboard constraint and that quite a few designs are already infeasible just because of this constraint. On the contrary, the majority of designs is quite far away from the draft constraint and only a small set encounters this constraint as active. A brief way to describe this is to consider the utilization index for the mean values: $u_{draft}\left(\frac{\mu_{draft}}{\sigma_{draft}}\right) = 5\%$ and $u_{freeboard}\left(\frac{\mu_{freeboard}}{\sigma_{freeboard}}\right) = 47\%$.

Any single design can now be judged by considering its utilization index in comparison to the mean values. A feasible design with $u_{freeboard} = 99\%$ would be one that lies very close to the freeboard constraint and considerably higher than the mean of 47%. A design with $u_{freeboard} = 101\%$, meanwhile, would be just about infeasible. Such a quantification would not be possible without normalization.

Finally, the relationship between constraints and free variables ought to be looked at. Fig. 9 and Fig. 10 display scatter diagrams of the normalized constraints of draft and freeboard vs. the normalized free variables $\tilde{x}_1$, x_1 being the ferry's depth. (It should be noted that the ordinates of Fig. 9 and Fig. 10 were the abscissas of Fig. 7 and Fig. 8, respectively.) Of course, all feasible designs lie below the line $\frac{g_k(\vec{x})}{\sigma_k} = 0$. Moreover, the general trend is that for higher $\tilde{x}_1$ the designs' mean get closer to the constraints. Moreover, for small $\tilde{x}_1$ neither the draft nor the freeboard constraint is crucial. In Fig. 10 it is found that the frontier of the feasible designs bend away from the line $\frac{g_k(\vec{x})}{\sigma_k} = 0$ for small $\tilde{x}_1$. In this region one or several other constraints are active. If a region of a free variable did not yield any feasible designs at all it could as well be discarded so as to subsequently reduce the search domain.

Fig. 11 and Fig. 12 represent similar plots for another free variable, i.e., $\tilde{x}_3$, x_3 being the parallel extent of a chine close to the design waterline. Again trends are noticeable which assist the design team in the constraint management.

Conclusion

Tangible improvement of the design process can be realized if as many constraints as possible are managed directly by the designer. This includes simultaneous screening of constraints, systematic assessment of interdependencies and support for decision taking. The theoretical background of constraint management stems from formal optimization. Several assessment instruments are proposed such as indices, frequency distributions and scatter plots.

A hydrodynamic design example was used to illustrate the constraint management approach taken within the consultancy work of the authors. A Sobol search strategy was employed in which many thousand variants had been automatically generated and evaluated even before time consuming further analyses were launched. The *FRIENDSHIP-Modeler* served to generate the hull forms and a very comprehensive picture could be drawn about the design task at hand. Focus was given on the management of geometric constraints. Nevertheless, non-geometric constraints can be treated equivalently.

Future work will focus on the further analysis and systematic interpretation of the feasible domain and the multitude of data generated when under-

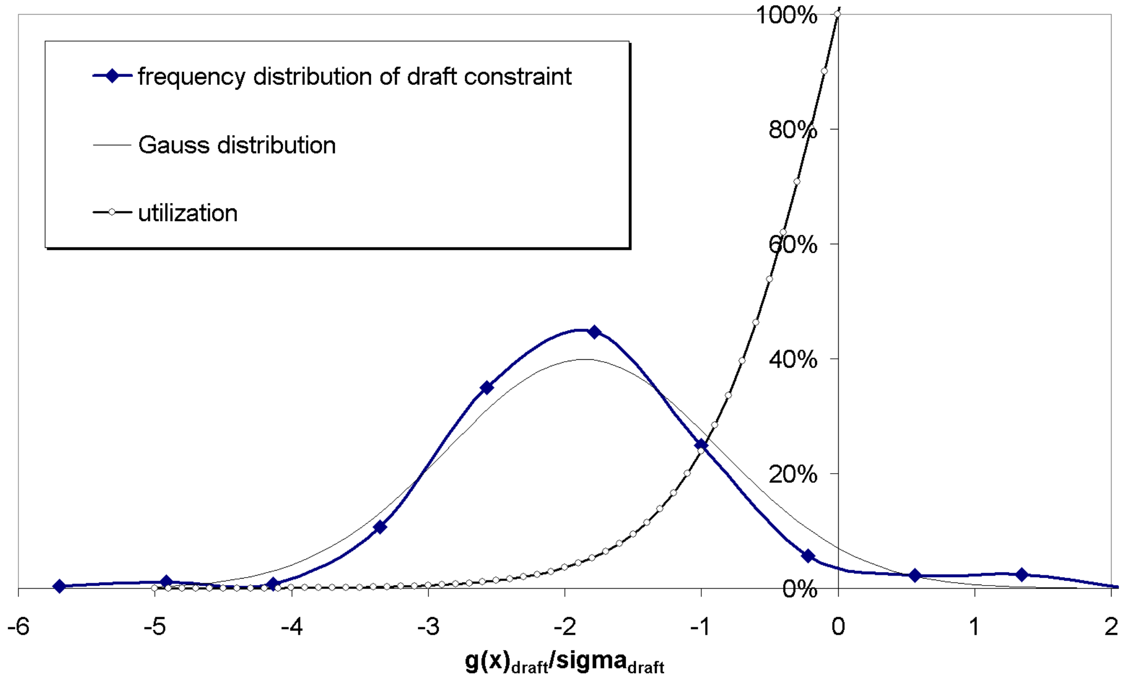

Figure 7: Frequency distribution of draft constraint

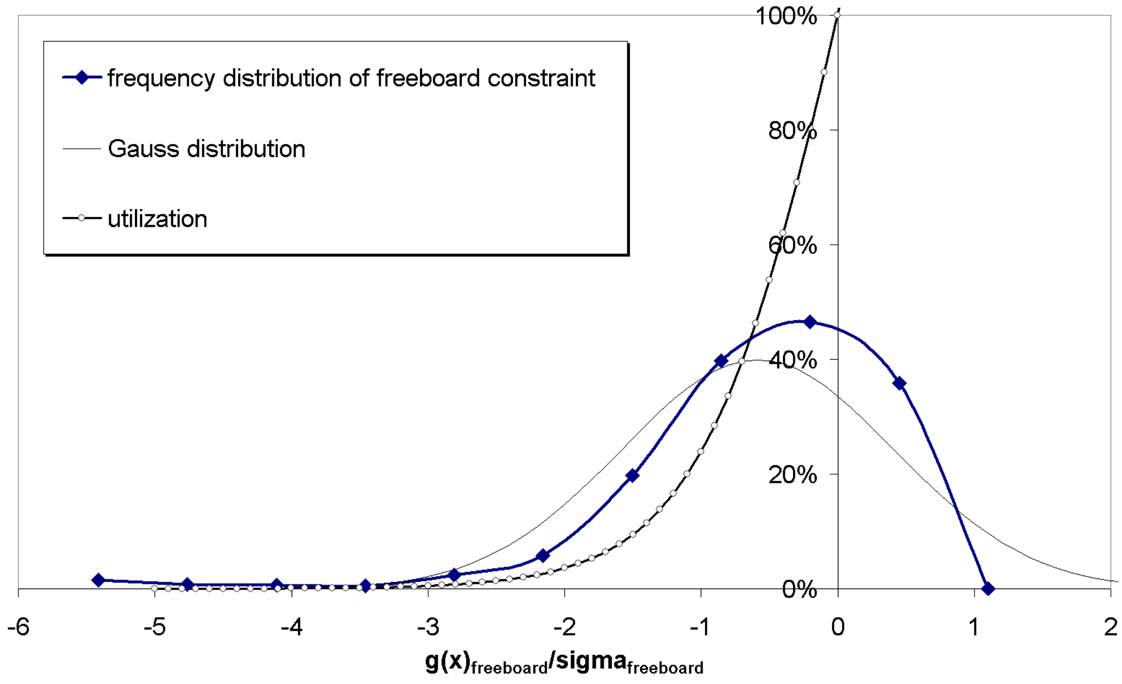

Figure 8: Frequency distribution of freeboard constraint

taking both (manual) design and (formal) optimization.

References

Abt, C., **Bade, S.**, **Birk, L.** and **Harries, S.** (2001): *Parametric Hull Form Design - A Step Towards One Week Ship Design, 8th International Sypmposium on Practical Design of Ships and Other Floating Structures, PRADS 2001*

Abt, C., **Harries, S.**, **Heimann, J.** and **Winter, H.** (2003): *From Redesign to Optimal Hull Lines by means of Parametric Modeling, 2nd International Conference on Computer Applications and Information Technology in the Maritime Industries*, Hamburg

Birk, L. and **Harries, S.** (Editors) (2003): *OPTIMISTIC – Optimization in Marine Design*, Mensch & Buch Verlag, Berlin, ISBN ISBN 3-89820-514-2

Harries, S. (1998): *Parametric design and hydrodynamic*

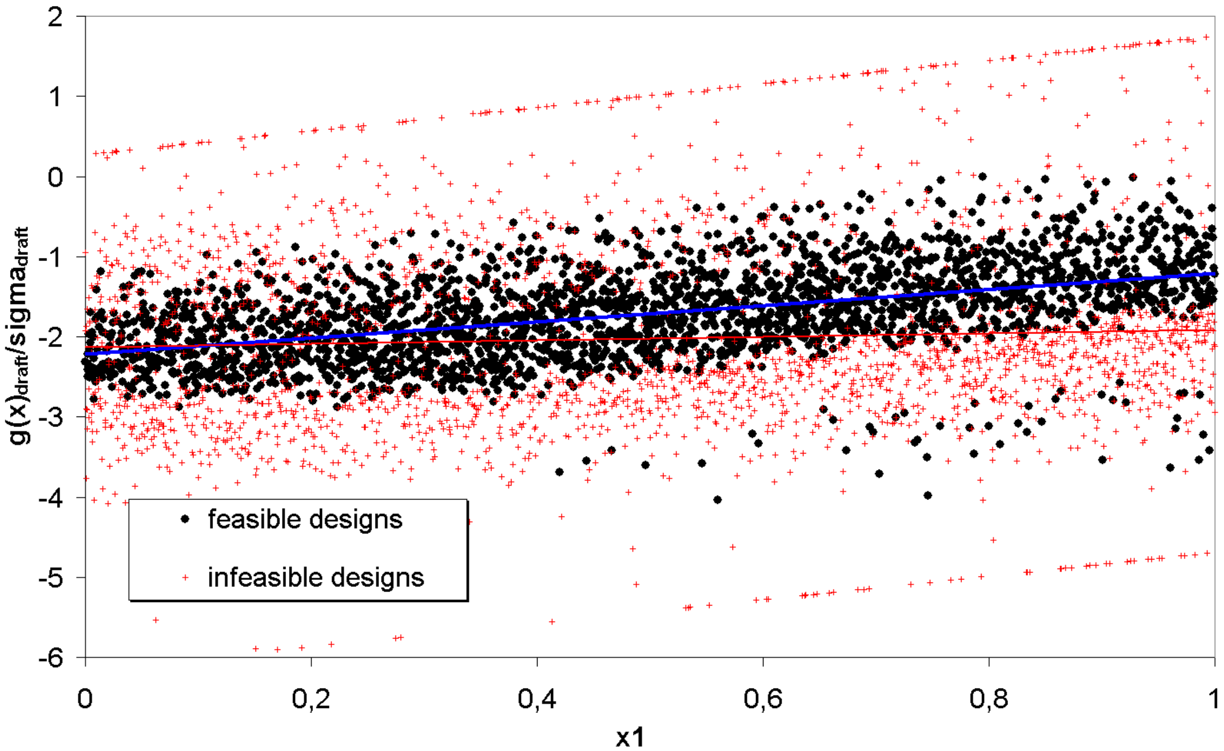

Figure 9: Scatter plot of draft constraint vs. free variable x_1

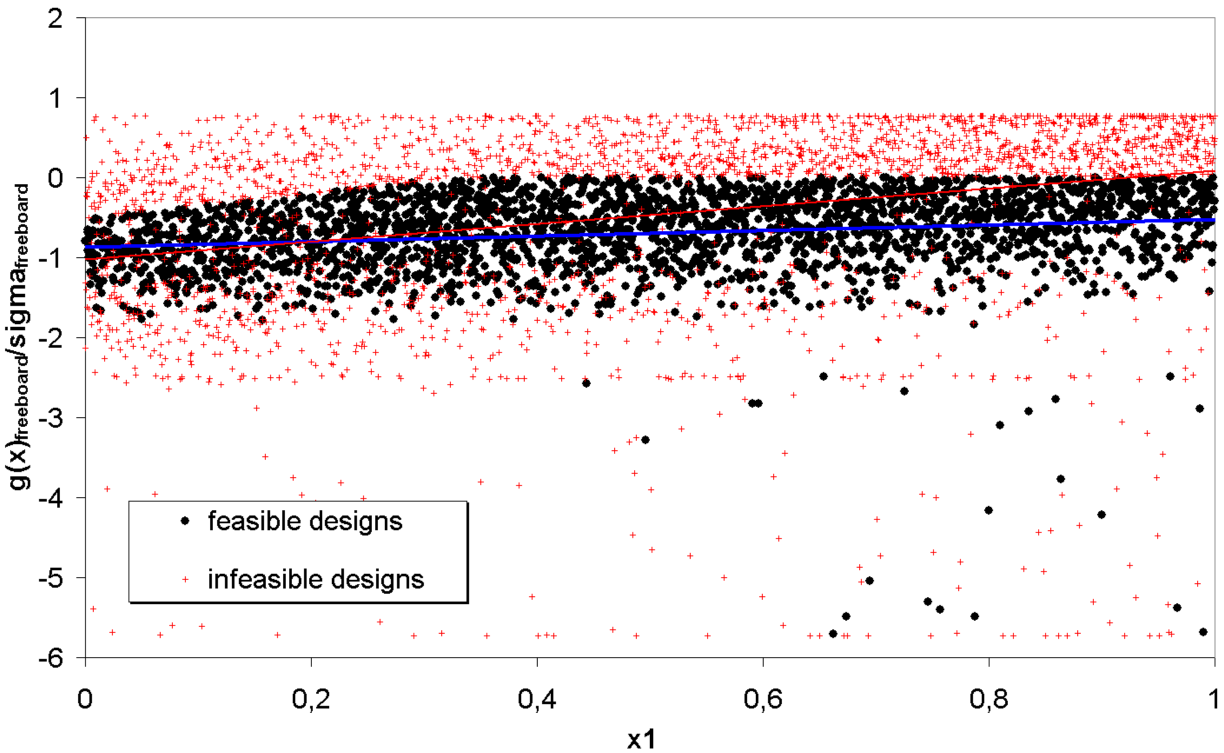

Figure 10: Scatter plot of freeboard constraint vs. free variable x_1

optimization of ship hull forms, Mensch & Buch Verlag, Berlin, ISBN 3-933346-24-X

Hochkirch, K., **Röder, K.**, **Abt, C.** and **Harries, S.** (2002): *Advanced parametric yacht design, High Performance Yacht Design Conference*, Auckland

Press, W. H., **Flannery, B. P.**, **Teukolsky, S. A.** and **Vetterling, W. T.** (1988): *Numerical recipies in C*, Cambridge University Press, New York

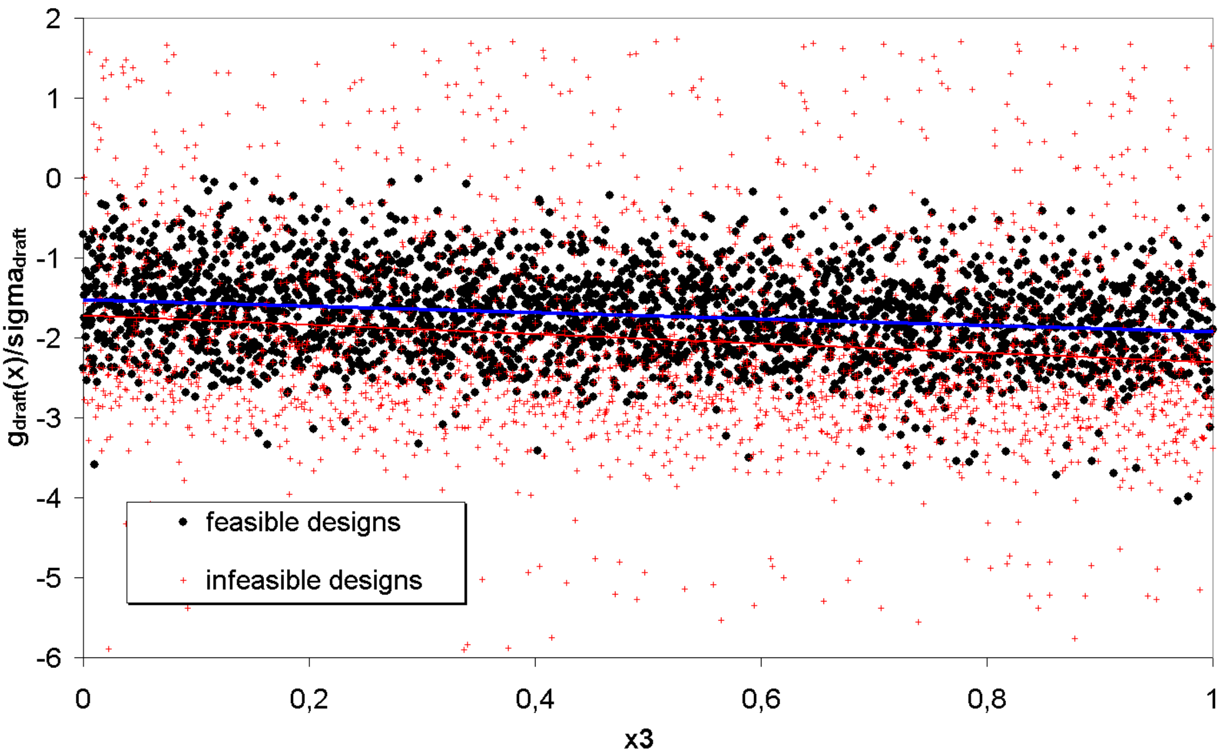

Figure 11: Scatter plot of draft constraint vs. free variable x_3

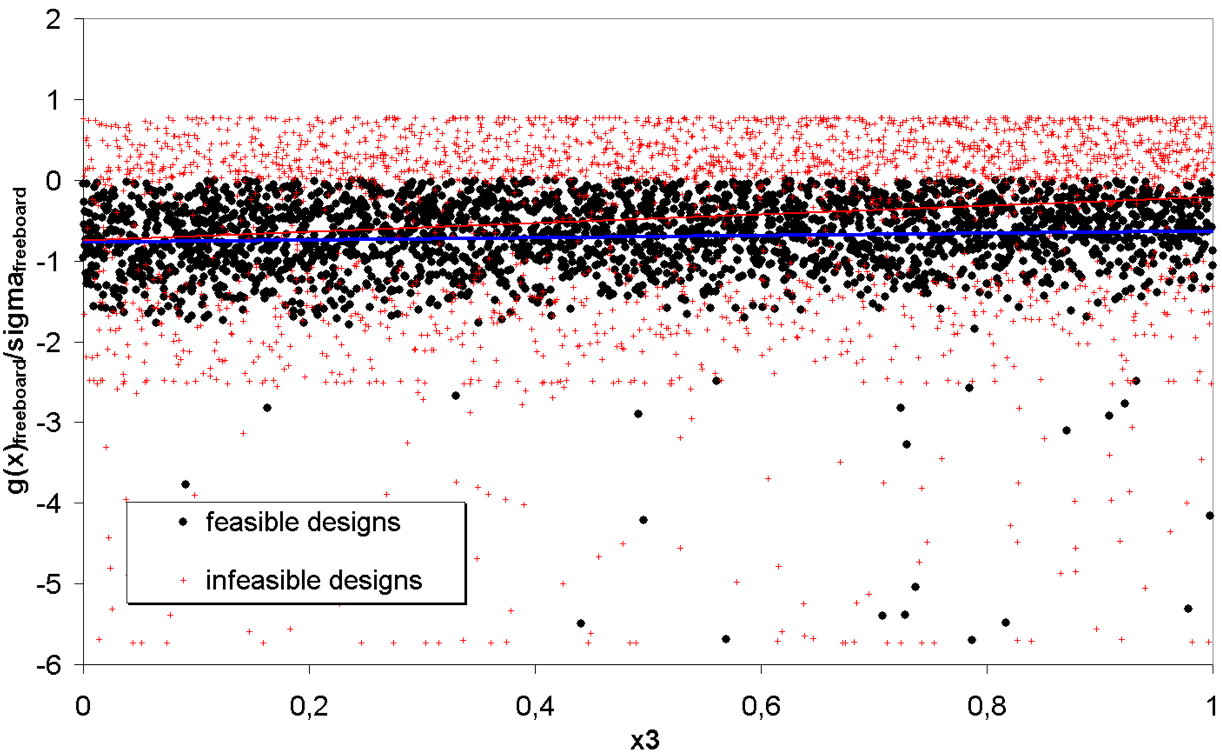

Figure 12: Scatter plot of freeboard constraint vs. free variable x_3